Chapter 11

Applied Measure

Learning Outcomes

In this chapter you will learn about:

- Area and perimeter of various shapes (review and extension from *Connect With Maths 1)*
- The nets of prisms, cylinders and cones
- The surface area of triangular-based prisms, cylinders and cones
- Solving problems relating to the curved surface area of cylinders, cones and spheres
- Calculations using formulae for the volume of rectangular solids, cylinders, cones, triangular-based prisms, spheres and combinations of these

Section 11A Triangles and Parallelograms

In this section we will build on our knowledge of perimeter and area of different shapes which we met in *Connect With Maths 1.*

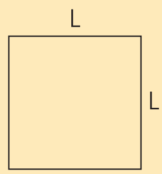
Key Points

The formulae for the perimeter and area of various shapes are summarised below.

- **Square**

  Area = length x length

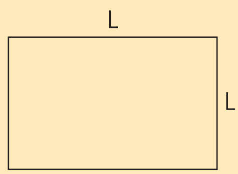
  Perimeter = 4 x length



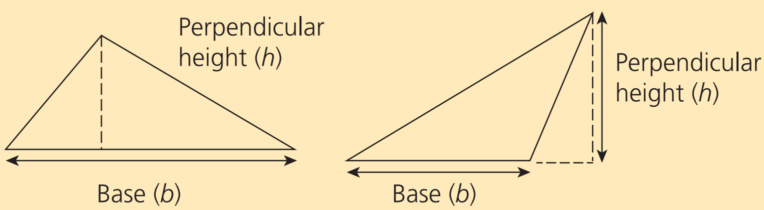
- **Rectangle**

  Area = length x width

  Perimeter = 2(length) x 2(width)



- **Triangle**



Area = $\frac{1}{2}$ x base (*b*) x perpendicular height (*h*)

Area = $\frac{1}{2}$ x b x h

Perimeter = sum of all three sides

Connections

We need to use our knowledge of geometry theorems in this section, especially that which relates to congruent triangles.

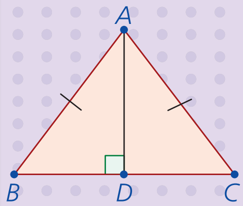
In the isosceles triangle ABC shown, $|AB| = |AC|$.

$[AD]$ is drawn perpendicular to $[BC]$.

The triangles ABD and ADC are congruent (RHS).

Hence $|BD| = |DC|$.

We will use this information in the following examples.

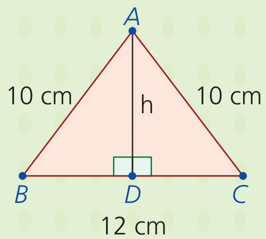


Example 1

ABC is an isosceles triangle with $|AB| = |AC| = 10$ cm and $|BC| = 12$ cm.

(i) Find h, the perpendicular height of the triangle ABC.

(ii) Find the area of ABC.

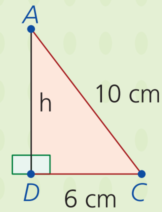


Solution

(i) The triangles ABD and ADC are congruent (RHS).

Hence, $|BD| = |DC| = 6$ cm.

Using the theorem of Pythagoras on triangle ADC, we have



$h^2 + 6^2 = 10^2$

$\Rightarrow h^2 = 10^2 - 6^2$

$\Rightarrow h^2 = 100 - 36 = 64$

$\Rightarrow h = \sqrt{64} = 8$ cm

(ii) Area of triangle ABC

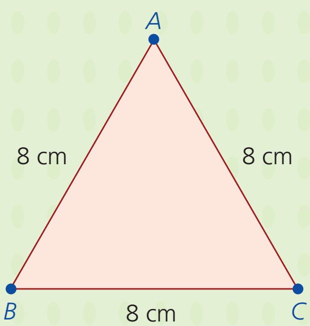
$\frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$= \frac{1}{2} \times 12 \times 8 = 48 \text{ cm}^2$

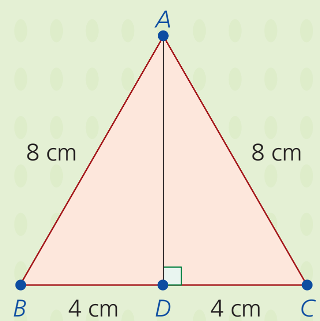
Example 2

ABC is an equilateral triangle of side length 8 cm.

Find the area of triangle ABC in surd form.



Solution



Draw $[AD] \perp [BC]$.

The triangles ABD and ADC are congruent (RHS).

Hence, $|BD| = |DC| = 4$ cm.

Using the theorem of Pythagoras on triangle ADC,

$h^2 + 4^2 = 8^2$

$\Rightarrow h^2 = 8^2 - 4^2$

$\Rightarrow h^2 = 64 - 16 = 48$

$\Rightarrow h = \sqrt{48} = 4\sqrt{3}$ cm

we have Area of triangle ABC

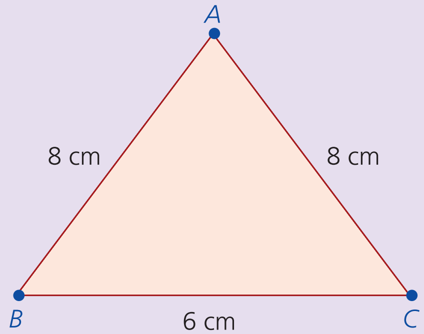
$= \frac{1}{2} \times$ base $\times$ perpendicular height

$= \frac{1}{2} \times 8 \times 4\sqrt{3} = 16\sqrt{3}$ cm^2

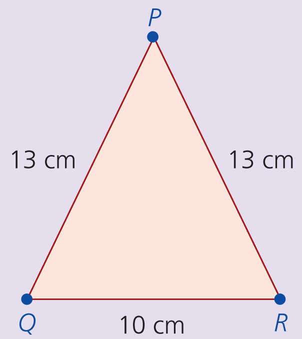
Activity 11.1

1 Find the area of each of the following isosceles triangles.

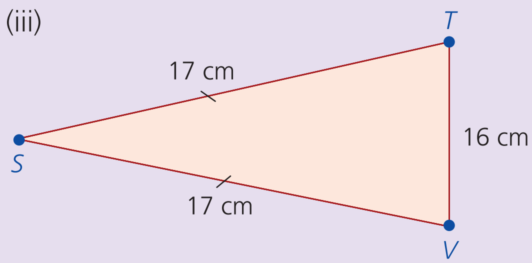
(i)



(ii)

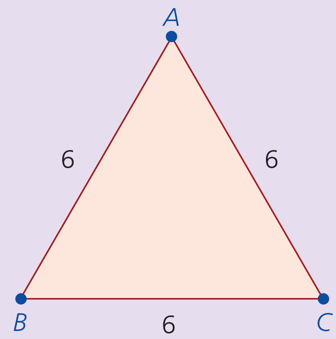


(iii)

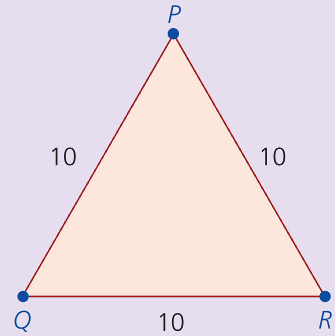


2 Find the area of the following equilateral triangles. Give your answers in surd form.

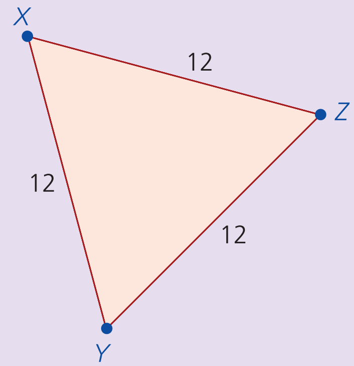
(i)



(ii)

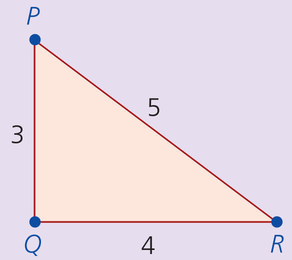


(iii)

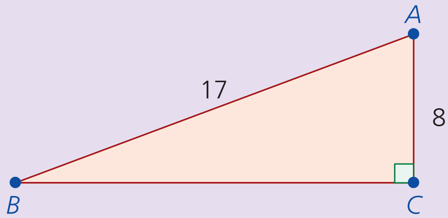


3 Find the area of the following right-angled triangles.

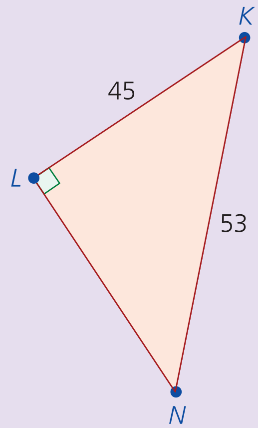
(i)



(ii)



(iii)

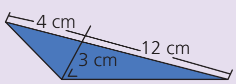


4 For each of the following triangles find:

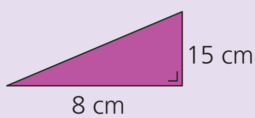
(i) the area of the triangle to the nearest cm²

(ii) the perimeter of the triangle (to the nearest cm in part (a)).

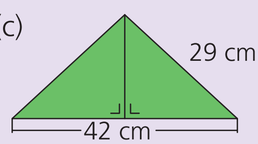
(a)



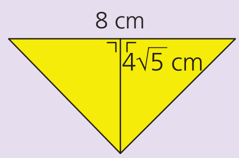
(b)



(c)



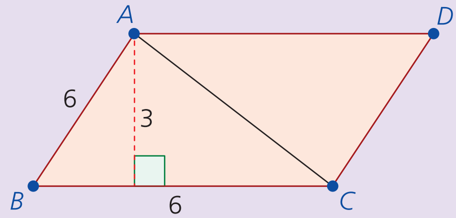
(d)



5 *ABCD* is a parallelogram with diagonal [*AC*]. |*AD*| = 6 and |*AB*| = 4. The perpendicular height = 3.

(i) Find the area of triangle *ABC*.

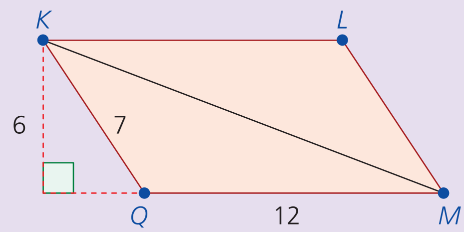
(ii) Prove that the triangles *ABC* and *ADC* are congruent.



(iii) How does the area of *ABCD* compare to the area of the triangle *ABC*?

(iv) Write down the area of parallelogram *ABCD*.

6 *KLMQ* is a parallelogram with diagonal [*KM*]. |*KQ*| = 7 and |*QM*| = 12. The perpendicular height = 6.



(i) Find the area of triangle *KQM*.

(ii) Show that the triangles *KQM* and *KLM* are congruent.

(iii) How does the area of *KLMQ* compare to the area of the triangle *KQM*?

(iv) Write down the area of parallelogram *KLMQ*.

(v) From your work in this question and also in question 5, can you give a formula for the area of a parallelogram?

The Area of a Parallelogram

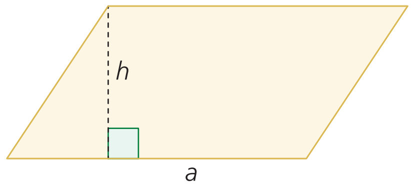
We saw in questions 5 and 6 of Activity 11.1 that the area of a parallelogram is twice that of a triangle on the same base with the same perpendicular height.

This is written as a formula as follows:

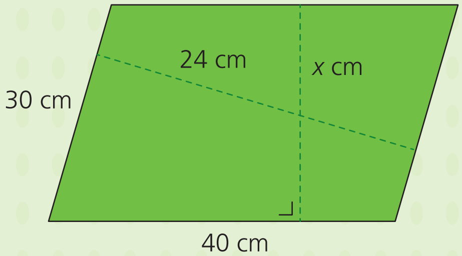
Area = base × perpendicular height

= a ×h

(See *Tables and formulae* page 8.)



Example 3



(i) Find the area of this parallelogram.

(ii) Hence or otherwise, find the value of the perpendicular height, x.

Solution

(i) Write down the information we know:

- Area of a parallelogram = base × perpendicular height = ah
- Base = a = 30 cm
- Perpendicular height = h = 24 cm

Calculate the area of the parallelogram:

Area = 30 × 24 = 720 cm²

(ii) Write down the information we know:

- Area of a parallelogram = ah = 720 cm2
- Base = a = 40 cm
- Perpendicular height = h = x cm

Calculate the perpendicular height (x) of the parallelogram:

- Area of parallelogram = base × perpendicular height

$\Rightarrow 40x = 720$

$\Rightarrow \quad x = \dfrac{720}{40}$ ...divide both sides by 40

$\Rightarrow \quad x = 18$ cm

Section 11B Arcs and Sectors of Circles

In *Connect With Maths 1* we worked with the circle (disc) and its properties.

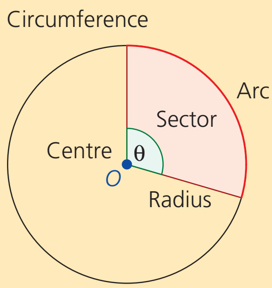
Key Points

Recall:

Circle:

Circumference of a circle = $2\pi r$, where r is the radius.

Area of a circle (disc) = πr^2



In some situations it is necessary to work out the perimeter and/or the area of a **sector** of a circle instead of the circumference/area of a full circle. For example sectors are used in:

- Pie charts 
- Window design 
- Garden design